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AVERAGING OF THE FLUCTUATIONS OF A SPHERICAL WAVE OVER THE RECEIVING APERTURE

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30 January 1973

FTD-HT-23-1

FOREIGN TECHNOLOGY DIVISION

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by

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UNCLASSIFIED

Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) Foreign Technology Division Air Force Systems Command U. S. Air Force	2a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED
	2b. GROUP
3. REPORT TITLE AVERAGING OF THE FLUCTUATIONS OF A SPHERICAL WAVE OVER THE RECEIVING APERTURE	
4. DESCRIPTIVE NOTES (Type of report and inclusive dates) Translation	
5. AUTHOR(S) (First name, middle initial, last name) A. I. Kon	

6. REPORT DATE 1969	7a. TOTAL NO. OF PAGES ~~6~~ 11	7b. NO. OF REFS 4
8a. CONTRACT OR GRANT NO.	9a. ORIGINATOR'S REPORT NUMBER(S) FTD-HT-23-1862-72	
b. PROJECT NO. 1279		
c.	9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)	
d.		

10. DISTRIBUTION STATEMENT

Approved for public release; distribution unlimited.

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY Foreign Technology Division Wright-Patterson AFB, Ohio

13. ABSTRACT

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FTD-HT-23-1862-72

EDITED TRANSLATION

FTD-HT-23-1862-72

AVERAGING OF THE FLUCTUATIONS OF A SPHERICAL WAVE OVER THE RECEIVING APERTURE

By: A. I. Kon

English pages: 6

Source: Radiofizika, Vol. 12, No. 1, 1969, pp. 149-152.

Requester: RADC

Translated by: TSgt Victor Mesenzeff

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Block	Italic	Transliteration	Block	Italic	Transliteration
А а	*А а*	A, a	Р р	*Р р*	R, r
Б б	*Б б*	B, b	С с	*С с*	S, s
В в	*В в*	V, v	Т т	*Т т*	T, t
Г г	*Г г*	G, g	У у	*У у*	U, u
Д д	*Д д*	D, d	Ф ф	*Ф ф*	F, f
Е е	*Е е*	Ye, ye; E, e*	Х х	*Х х*	Kh, kh
Ж ж	*Ж ж*	Zh, zh	Ц ц	*Ц ц*	Ts, ts
З з	*З з*	Z, z	Ч ч	*Ч ч*	Ch, ch
И и	*И и*	I, i	Ш ш	*Ш ш*	Sh, sh
Й й	*Й й*	Y, y	Щ щ	*Щ щ*	Shch, shch
К к	*К к*	K, k	Ъ ъ	*Ъ ъ*	"
Л л	*Л л*	L, l	Ы ы	*Ы ы*	Y, y
М м	*М м*	M, m	Ь ь	*Ь ь*	'
Н н	*Н н*	N, n	Э э	*Э э*	E, e
О о	*О о*	O, o	Ю ю	*Ю ю*	Yu, yu
П п	*П п*	P, p	Я я	*Я я*	Ya, ya

* ye initially, after vowels, and after ъ, ь; e elsewhere. When written as ё in Russian, transliterate as yё or ё. The use of diacritical marks is preferred, but such marks may be omitted when expediency dictates.

FOLLOWING ARE THE CORRESPONDING RUSSIAN AND ENGLISH DESIGNATIONS OF THE TRIGONOMETRIC FUNCTIONS

Russian	English
sin	sin
cos	cos
tg	tan
ctg	cot
sec	sec
cosec	csc
sh	sinh
ch	cosh
th	tanh
cth	coth
sch	sech
csch	csch
arc sin	$\sin^{-1}$
arc cos	$\cos^{-1}$
arc tg	$\tan^{-1}$
arc ctg	$\cot^{-1}$
arc sec	$\sec^{-1}$
arc cosec	$\csc^{-1}$
arc sh	$\sinh^{-1}$
arc ch	$\cosh^{-1}$
arc th	$\tanh^{-1}$
arc cth	$\coth^{-1}$
arc sch	sech^{-1}
arc csch	csch^{-1}
rot	curl
lg	log

AVERAGING OF THE FLUCTUATIONS OF A SPHERICAL WAVE OVER THE RECEIVING APERTURE

A. I. Kon

If a light source is in a turbulent medium or at a short distance from a turbulent layer, then the incident wave cannot be considered as plane, and in all calculations it is necessary to consider its spherical property. The problem which is of practical interest, dealing with the incidence of a wave which passed through a turbulent layer on an elongated objective, was solved in [1] only for a plane wave. In this work the results of [1] are generalized for the case of a spherical wave.

Let us assume that a spherical wave, after traveling path L from a light source located at distance x_0 from a turbulent layer through a nonhomogeneous medium, is incident on a circular objective with area $\Sigma = \pi R^2$. If I(y, z) is the incident wave intensity in the receiver's plane, then the total light flux through the objective is equal to

$$P = \iint_{\Sigma} I(y, z)\, dy\, dz. \qquad (1)$$

Designating $I' = I - \langle I \rangle$, for fluctuations $P' = P - \langle P \rangle$ we have

$$P' = \iint_{\Sigma} I'(y, z)\, dy\, dz. \tag{2}$$

The mean square of fluctuations of the total light flux is determined by the formula

$$\langle P'^2 \rangle = \iint_{\Sigma} \iint_{\Sigma} B_I(y_1, z_1, y_2, z_2)\, dy_1\, dz_1\, dy_2\, dz_2, \tag{3}$$

where $B_I(y_1, z_1, y_2, z_2) = \langle I'(y_1, z_1)\, I'(y_1, z_2) \rangle$. Light intensity fluctuations of a spherical wave, generally speaking, are nonhomogeneous in the plane of the objective; however, since the longitudinal correlation radius has a length on the order of the path [2] it is easy to show that inside the cone with the angle of taper $\theta \ll 1$, the fluctuations in the objective's plane can be considered to be homogeneous with a high degree of accuracy. Consequently, $B_I(y_1, z_1, y_2, z_2) = B_I(y_1 - y_2, z_1 - z_2)$. It is clear that in practice, condition $\theta \ll 1$ is fulfilled quite well since the solid angle at which the receiving objective is visible from the source is usually small.

Using the homogeneity and isotropy of the correlation function of fluctuations of intensity B_I and reasoning in the way as in [1], the following formula can be obtained for function $G(R)$ which represents the ratio of value $F(R) = \langle P'^2 \rangle / \langle P \rangle^2$ of the elongated receiver to the same expression for the point receiver:

$$G(R) = \frac{4}{\pi R^2} \int_0^{2R} b_I(\rho) \left[\arccos\left(\frac{\rho}{2R}\right) - \frac{\rho}{2R} \sqrt{1 - \frac{\rho^2}{4R^2}} \right] \rho\, d\rho, \tag{4}$$

where $b_I(\rho) = B_I(\rho)/B_I(0)$ - correlation coefficient of fluctuations of intensity in a spherical wave. It is clear that value G(R) characterizes a decrease in relative fluctuations of the total light flux through the objective depending on its size.

Considering equality $I = A_0^2 \exp(2_\chi)$ where $\chi = \ln(A/A_0)$ and the fact that value χ is distributed normally, it is possible to obtain the following relationship for case $\langle\chi^2\rangle \ll 1$ (i.e., in the area where the method of even perturbations is applicable [3]):

$$b_I(\rho) = \frac{B_I(\rho)}{\langle\chi^2\rangle} \equiv b_\chi(\rho). \qquad (5)$$

Here $\langle\chi^2\rangle$ - mean square of fluctuations of the logarithm of the spherical wave amplitude; B_χ - correlation function of fluctuations of the amplitude logarithms in the plane perpendicular to the line connecting the source with the receiver. By means of (5) the formula for G(R) can be rewritten in the form

$$G(R) = \frac{4}{\pi R^2}\int_0^{2R} b_\chi(\rho)\left[\arccos\left(\frac{\rho}{2R}\right) - \frac{\rho}{2R}\sqrt{1-\frac{\rho^2}{4R^2}}\right]\rho\, d\rho. \qquad (6)$$

To calculate correlation coefficient $b_\chi(\rho)$ we can use the correlational functions of fluctuations of the complex phase of spherical wave $\Psi_1 = \chi + iS_1$ ($S_1 = S - \langle S\rangle$ - fluctuations of the actual phase). For the correlational functions of the complex phase, we have[1]

[1]Formulas for the correlational functions of the complex phase were given to the author by Yu. A. Kravtsov and Z. I. Fryzulin.

$$B_{\psi\psi} = \langle \Psi_1(x_0 + L, \rho_1) \Psi_1(x_0 + L, \rho_2) \rangle = -4\pi^2 k^2 \int\limits_{x_0}^{x_0+L} dx \int\limits_0^\infty d\varkappa \times$$
$$\times \varkappa \Phi_n(\varkappa) \exp\left[-\frac{i\varkappa^2 x(x_0 + L - x)}{k(x_0 + L)}\right] J_0\left(\frac{\varkappa x}{x_0+L} |\rho_1 - \rho_2|\right); \tag{7}$$

$$B_{\psi\psi^*} = \langle \Psi_1(x_0 + L, \rho_1) \Psi_1^*(x_0 + L, \rho_2) \rangle = +4\pi^2 k^2 \int\limits_{x_0}^{x_0+L} dx \int\limits_0^\infty d\varkappa \times$$
$$\times \varkappa \Phi_n(\varkappa) J_0\left(\frac{\varkappa x}{x_0+L} |\rho_1 - \rho_2|\right). \tag{8}$$

Here $J_0(x)$ - Bessel function, $\Phi_n(\kappa)$ - three-dimensional spectrum of fluctuations of the refractive index.

Using the obvious relationship for correlational function $B_\chi = \mathrm{Re}(B_{\psi\psi} + B_{\psi\psi*})/2$, it is easy to find

$$B_\chi(|\rho_1 - \rho_2|) \equiv B_\chi(\rho) = 2\pi^2 k^2 \int\limits_{x_0}^{x_0+L} dx \int\limits_0^\infty \Phi_n(\varkappa) J_0\left(\frac{\varkappa x}{x_0+L}\rho\right) \times$$
$$\times \left[1 - \cos\frac{\varkappa^2 x(x_0 + L - x)}{k(x_0 + L)}\right] \varkappa d\varkappa \tag{9}$$

Substituting correlational function (9) into expression (6), for function G(R) we obtain

$$G(R) = \frac{8\pi k^2}{R^2 \langle \chi^2 \rangle} \int\limits_0^{2R} \left[\arccos\left(\frac{\rho}{2R}\right) - \frac{\rho}{2R}\sqrt{1 - \frac{\rho^2}{4R^2}}\right] \rho d\rho \times$$
$$\times \int\limits_{x_0}^{x_0+L} dx \int\limits_0^\infty \Phi_n(\varkappa)\left[1 - \cos\frac{\varkappa^2 x(x_0 + L - x)}{k(x_0 + L)}\right] J_0\left(\frac{\varkappa x}{x_0+L}\rho\right) \varkappa d\varkappa, \tag{10}$$

where the mean square of fluctuations of the logarithm of a spherical wave is

$$\langle \chi^2 \rangle = 2\pi^2 k^2 \int\limits_{x_0}^{x_0+L} dx \int\limits_0^\infty \Phi_n(\varkappa)\left[1 - \cos\frac{\varkappa^2 x(x_0 + L - x)}{k(x_0 + L)}\right] \varkappa d\varkappa. \tag{11}$$

Changing the order of integration in (10), it is possible to calculate the integral for ρ [4]:

$$G(R)=\frac{8\pi^2k^2(x_0+L)^2}{R^2\langle\chi^2\rangle}\int\limits_{x_0}^{x_0+L}\frac{dx}{x^2}\int\limits_0^\infty\frac{d\varkappa}{\varkappa}\left[1-\cos\frac{\varkappa^2x(x_0+L-x)}{k(x_0+L)}\right]\times$$
$$\times J_1^2\left(\frac{\varkappa Rx}{x_0+L}\right)\Phi_n(\varkappa). \tag{12}$$

After the substitution of spectrum $\Phi_n(\kappa) = AC_n^2\ \chi^{-11/3}$ corresponding to the "law of 2/3" and transition to the dimensionless integration variables $\xi = x/L$ and $y = \kappa^2 L/k$, expression (15) assumes the form

$$G(R)=\frac{4\pi^2AC_n^2k^{1/6}L^{17/6}\left(1+\frac{x_0}{L}\right)^2}{R^2\langle\chi^2\rangle}\int\limits_{x_0/L}^{1+x_0/L}d\xi\int\limits_0^\infty\xi^{-2}y^{-17/6}\times$$
$$\times\left[1-\cos\frac{\xi y(1+x_0/L-\xi)}{1+x_0/L}\right]J_1^2\left(\frac{k^{1/2}R}{L^{1/2}}\frac{\xi y^{1/2}}{1+x_0/L}\right)dy. \tag{13}$$

Using the limiting transition $x_0 \to \infty$, it is possible to obtain an appropriate formula from formula (13) for a plane wave; in this case the indicated limiting transition should also be realized in expression (14) for $\langle\chi^2\rangle$.

Let us examine case $x_0 = 0$ in more detail (the source is in a turbulent medium). Using (11) and (13), in this case, the following formula can be obtained for function G(R):

$$G(R)=\frac{8\Gamma(11/3)\cos(\pi/12)}{\pi\Gamma(11/6)\alpha_R^2}\int\limits_0^1d\xi\int\limits_0^\infty y^{-17/6}\frac{1-\cos[y\xi(1-\xi)]}{\xi^2}J_1^2(\alpha_R\xi y^{1/2})\,dy, \tag{14}$$

where $\Gamma(x)$ - gamma-function, $\alpha_R = k^{1/2}R/L^{1/2}$ - value characterizing the size of the receiving objective as compared to the radius of

the first Fresnel zone for a plane wave at a given distance L.

The results of numerical calculation for function G(R) which describes the averaging effect of the objective on the fluctuations of a spherical wave are given in Fig. 1.

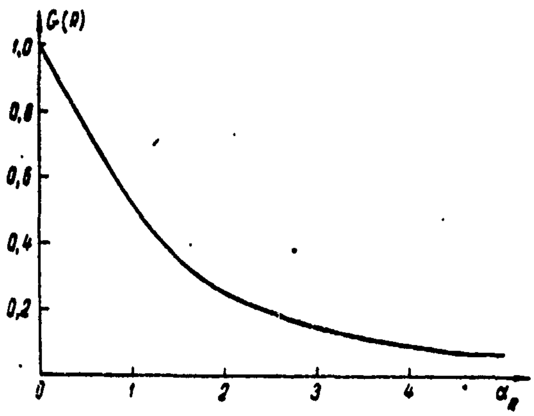

Fig. 1. Function describing the averaging of fluctuations in a spherical wave over the receiver aperture

$\alpha_R = k^{1/2} R/L^{1/2}$).

The author expresses his appreciation to V. I. Tatarskiy for his valuable opinions and help.

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Received
22 Apr 68